TITLE: SELECTING THE OPTIMAL COMBINATION OF MEMBERSHIP FUNCTIONS FOR QUERIES BASED ON FUZZY LOGIC BY EVALUATING THE SENSITIVITY

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KEYWORDS

linguistic variable, membership function, fuzzy logic

ABSTRACT

Efficient request distribution is one of the important factors for the stable and uninterrupted operation of modern information systems. In the context of increasing cloud computing, real-time services and multi-user environments, traditional distribution algorithms (e.g. Round Robin, Least Connection, Weighted Response Time) are losing their effectiveness due to insufficient consideration of uncertainty and dynamic changes in server state. In this context, there is a growing need for decision-making systems based on adaptive and state-sensitive approaches.

Fuzzy logic-based models have a significant advantage in representing uncertainties in real-world environments, especially due to the ability to estimate system parameters through linguistic variables. However, the effectiveness of these systems directly depends on the shape and parameters of the membership functions, i.e. their semantic relevance and sensitivity. Using membership functions of the same shape for each linguistic value (for example, only a triangle) can reduce the accuracy of the system's decision-making

Introduction. The current state of load balancing is characterized by a wide range of methods and technologies used to evenly distribute traffic across multiple servers, ensuring high availability, scalability, and application performance [1]. A literature review reveals various approaches, including hardware and software load balancers, as well as the use of cloud services [2].

In the proposed approach, appropriate membership functions are selected based on semantic meaning for parameters representing server state, their sensitivity levels are evaluated, and a balanced request distribution over computing resources is implemented by forming hybrid fuzzy logic decision models. In traditional approaches, the use of membership functions of the same shape (for example, a triangle or a trapezoid) for all linguistic variables reduces sensitivity and negatively affects the accuracy and stability of system results [1][3]. Therefore, the use of combination functions adapted to parameter semantics plays an important role in increasing system sensitivity.

Studies have shown that even small parameter changes in the membership functions of NMTs can have a significant impact on the output results [3][4]. This makes sensitivity analysis an indispensable step in improving the efficiency of NMTs. Methodologies such as fuzzy regression, nonlinear regression, Morris screening have been used as effective tools in this process [1][2][4]. Also, the activity of input parameters and their influence on decision-making based on adaptive neural-fuzzy systems have been studied in depth [5]. Using these methods, in-depth sensitivity analysis allows to increase the system's adaptability to the situation in real time and ensure efficient use of resources.

The main advantage of the NM approach is the ability to assess the server status using linguistic variables, i.e. human expressions (e.g., "high load", "slow response time", "medium power") and create an appropriate allocation mechanism (Table 1).

Table 1

Comparative analysis of classical and fuzzy logic approaches

Type of approach	Adaptability to the situation	Sensitivity to system changes	Computing power	Accuracy

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Round Robin	No	Low	Very low	Medium
Least Connection	Medium	Medium	Medium	Medium
Fuzzy based model	High	High	Medium / High	High

Methodology

Sensitivity assessment of relevance functions. A hybrid approach model has been developed to optimize the overall sensitivity of the system, which allows selecting relevance functions that correspond to the natural characteristics of each linguistic state in the distribution of questions based on fuzzy logic by quantifying the sensitivity parameters. This model allows to increase the efficiency of decision-making by optimizing the overall sensitivity of the system and minimize the uncertainties that arise in the system. Determining the sensitivity level of relevance functions corresponding to each linguistic value state using a combinatorial approach ensures optimal system performance[10].

The proposed combinational approach, i.e., applying triangular, trapezoidal, and Gaussian functions to each server state, aims to provide optimal sensitivity for servers. The triangular function is used for low load conditions, the trapezoidal function for medium load conditions, and the Gaussian function for high load conditions. Each sensitivity function, according to its characteristics, provides an optimal result for each system state. The main goal of this approach is to effectively control the system based on the parameters of each server (for example, load level, response time, access to resources, and their performance).

In this combined approach, the triangular function helps to identify low sensitivity, i.e., very low load conditions of the system. The trapezoidal function shows medium sensitivity and describes stable operating conditions of the system. The Gaussian function is used to identify high sensitivity, i.e., conditions where the system is under high load. In the process of calculating the specific sensitivity parameters of each function, all of them are optimized for different system conditions.

The analysis steps of this combinatorial approach are presented in Figure 1. It clearly defines the sensitivity levels for each of the triangular, trapezoidal, and Gaussian functions and shows how they work to ensure the efficiency of the system in each case. This diagram helps to clearly understand which approach to use to select the optimal sensitivity for each system condition.

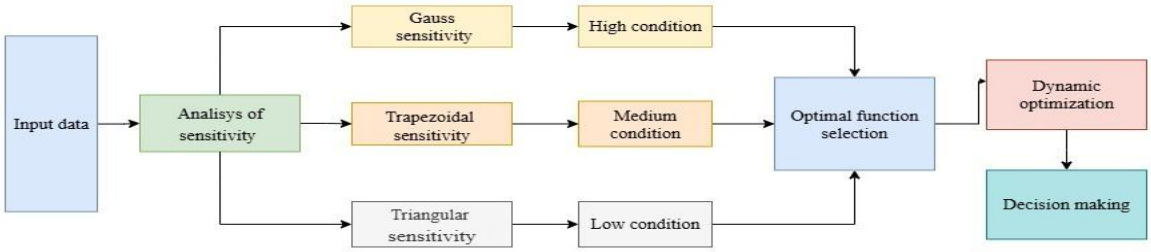

Figure 1. Model for optimizing the sensitivity properties of membership functions in server load analysis

This approach helps to improve the sensitivity of the membership functions to the state of each linguistic variable, thereby optimizing the overall performance of the system.

The overall sensitivity is determined by optimizing the combinatorial states of the membership functions, which is expressed as:

$$S_{kombinatsiya} = \arg\min_{f_i \in F} \left[\sum_{k=1}^{3} w_k \cdot \delta S_k(f_k) \right], \quad (1)$$

where f is the set of available functions, such as triangular, trapezoidal, Gaussian membership functions; δ is the sensitivity to the k-state mismatch, i.e. the degree of influence of the input values on the output results; w_k is the state

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weight, i.e. the role and influence of each state in the system in decision-making.

The sensitivity of membership functions plays a key role in managing uncertainty in a system. Thus, an optimized approach to sensitivity estimation can significantly improve the efficiency of decision-making in fuzzy systems.

The combinational approach uses the following steps to ensure optimal sensitivity for each situation:

1. **Function set selection.** Three basic relevance functions (triangular, trapezoidal, Gaussian) are selected for each linguistic value. These functions increase the flexibility of the system to different situations.
2. **Sensitivity assessment.** For each case, the sensitivity discrepancy level $\delta S_k(f_k)$ is determined. At this stage, the influence of uncertainty and parameters in the fuzzy system is measured.
3. **Optimal combination.** For each case, the weight w_k is taken into account and the optimal decision is made to minimize the combined sensitivity. In this step, the optimal combination of membership functions is determined to improve the efficiency of the system

Feature selection based on sensitivity metrics

Relevance functions are important for the efficient organization of the decision-making process. In particular, through the combination of a set of rules, in some cases, the same value of the result can be observed. These situations arise, in particular, as a result of the use of the same relevance function, that is, there is a possibility of a decrease in sensitivity or a lack of smoothness between cases with the same linguistic value. In this regard, it is very important to determine the correct choice and combination of relevance functions for each case of the system. This approach allows to increase the overall efficiency of the system, ensure accuracy and minimize the uncertainties that arise in the decision-making process.

Results and Analisis

In this work, the aim is to increase the sensitivity of the system by selecting the appropriate combinational relevance function. For this purpose, methods for assessing the sensitivity of relevance functions and analyzing their results are used. First, we consider the relevance function of the low load state, which is one of the linguistic variables of servers.

The condition that requires the system to keep the response time to a minimum is the low load condition. In this case, the system requires fast response and high sensitivity. High sensitivity allows the system to process incoming requests efficiently and quickly. Thus, for low load conditions, it is necessary to provide sharp sensitivity of the system. Sharp sensitivity represents the speed of change between input values and output results.

To assess the sensitivity of a given parameter, the maximum sensitivity metric is used. This metric shows how the sensitivity function is affected by the parametric variation. The maximum sensitivity is expressed as:

$$S_{max}^{triangular\ max(\frac{1}{b-ac-b}} \qquad (2)$$

where $\boldsymbol{a}$ is the starting point (lower limit) of the triangular membership function; $\boldsymbol{b}$ is the midpoint (middle value of the feature); $\boldsymbol{c}$ is the high point (upper limit); $\boldsymbol{S_{max}}$ is the maximum sensitivity, i.e., the maximum degree of change of the function

In this formula, the parameters **a, b,** and **c** represent the geometric dimensions of the triangle function. Changing these parameters directly affects the level of sensitivity of the system. The parameter $\boldsymbol{S_{max}}$ ensures that the system has a sharp response and achieves maximum sensitivity (Figure 2).

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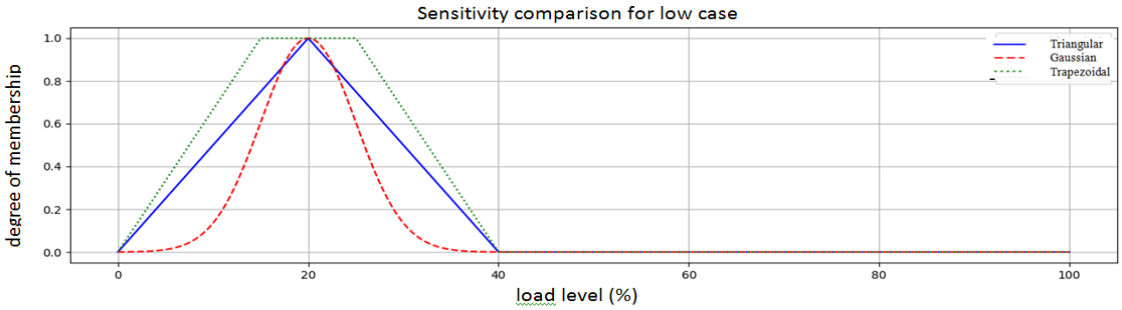

Figure 2. Sensitivity result for low load condition

The graphical results show that the **Gaussian** and **trapezoidal** membership functions have smaller values of S_{max} (Table 2). This is clearly seen in the image representing the change in the sensitivity levels of these functions. The triangular membership function showed the highest results in providing maximum sensitivity at low load. At the same time, the **Gaussian** and **trapezoidal** functions provide significantly lower values in sensitivity control, which limits the rapid response of the system.

Table 2

Results of comparison of membership functions based on the S_{max} metric

Type of	S_{max}	S_D	Note
Triangular	0.125	0.118 ±	High
Gaussian	0.084	0.076 ±	Low
Trapezoidal	0.091	0.087 ± 0.004	Medium sensitivity

The analysis results show that the optimality of each parameter plays an important role in ensuring the efficient operation of the system. For example, high sensitivity allows for a fast response of the system and provides the most efficient solution for low load conditions.

The medium load case requires system stability, because in the medium case, systems do not operate under very high or very low loads. For the stable operation of the system, it is necessary to ensure medium sensitivity. For this, the integral sensitivity metric was used. The trapezoidal function is the optimal choice for the medium load case, because it provides the stability and sensitivity of the system at a smooth and moderate level.

The integral sensitivity of the trapezoidal function is calculated as follows:

$$S_I^{trapezoidal} = \int_a^d \left|\frac{d\mu}{dx}\right| \rightleftarrows dx = \frac{1}{b-a} + \frac{1}{d-c}, \quad (3)$$

where ***a*** is the lower left point of the trapezoid function, i.e. the starting point of the sensitivity; b is the upper left point of the trapezoid function, i.e. the beginning of the change in sensitivity; c is the lower right point of the trapezoid function, i.e. the unchanged state of the sensitivity; d is the upper right point of the trapezoid function, i.e. the end point of the sensitivity; S_I is the integral sensitivity metric, i.e. the measurement of the sensitivity of the system over the entire interval.

The trapezoidal function is effective in balancing the sensitivity for systems in the medium state, helping to stabilize the sensitivity level in each state of the system. This metric plays an important role in ensuring a high level of system performance. The integral sensitivity of the trapezoidal function increases the stability of the overall system management, while balancing the sensitivity to changes in the system (Figure 3).

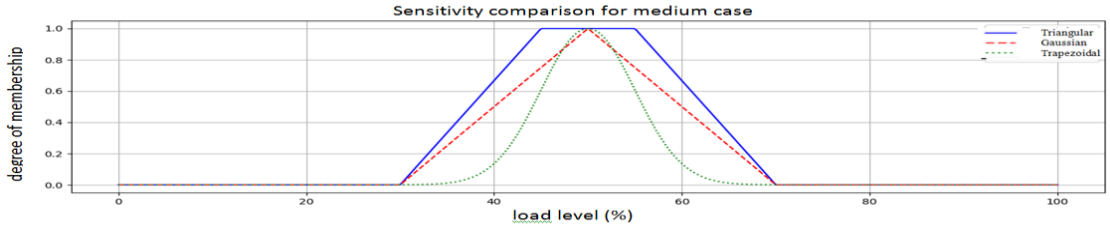

Figure 3. Graphical representation of sensitivity for the medium case

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The results of the graph show that the **Triangular** and **Gaussian** membership functions have smaller values of S_I (Table 3). This is clearly seen in the image representing the change in the sensitivity levels of these functions. In addition, the trapezoidal function has a high stability index and controls the system with smooth sensitivity. Compared with other functions, the trapezoidal function ensures stable operation of the system. As can be seen from the graph, the Trapezoidal function shows the highest results in providing maximum sensitivity at medium load. At the same time, the Triangular and Gaussian functions provide significantly lower values in sensitivity control, which limits the rapid response of the system.

Table 3

Results of comparison of membership functions based on S_I metrics

	Function	S_I	Sustainability Index	Note
1	Trapezoidal	1.8	0.82 (High)	High stability for medium conditions.
2	Gaussian	2.3	0.77 (Medium)	Optimal forecasting for high-state systems.
3	Triangular	2.0	0.80 (High)	Maximum sensitivity and fast response for low load conditions.

The high load state of the system requires forecasting and nuanced assessment. The concept of nuanced assessment refers to a precise and detailed study of each feature, change in the system and their impact on the system. To ensure accurate operation of the system under high load conditions, a high level of forecasting accuracy and decision-making efficiency are required. In this regard, it is important to accurately forecast and understand the changes in the system due to load, because under high load the system must perform fast and reliable calculations to make quick and correct decisions.

To improve the performance of the system in high-load conditions and ensure the accuracy of the forecast, the root mean square sensitivity metric is used. The root mean square (RMS) sensitivity metric provides an excellent estimate of the sensitivity of the system, since this metric allows you to measure the overall response of the system, that is, the average of the input parameters and the output results. In this case, the sensitivity of the system allows you to determine not only the performance under high load conditions, but also to analyze how the system responds to unexpected changes.

This metric helps in determining each feature of the system, because accurate prediction of system performance under high load ensures efficient system operation. At the same time, it is necessary to keep the RMS sensitivity at a minimum in the high load condition, because this stabilizes the system performance and provides the system with high prediction accuracy. Therefore, the RMS metric is the most effective and reliable evaluation tool for high load conditions.

In the upper case, the root-mean-square sensitivity metric is used to evaluate the system. This metric can be used to calculate the sensitivity of the system using the Gaussian function as follows:

$$S_{RMS}^{Gaussian} = \frac{1}{b-a}\int_a^b \left(\frac{x-c}{\sigma^2} e^{-\frac{(x-c)^2}{2\sigma^2}}\right)^2 \rightleftarrows dx, \quad (4)$$

where **x** is the input parameter; **c** is the center (average value) of the Gaussian function; **σ** is the variance (function width); $\boldsymbol{S_{RMS}}$ is the root mean square sensitivity.

The root mean square sensitivity allows us to assess not only the system's predictive ability but also its performance under high load conditions. This metric allows us to assess the system's fast response and the structure under high load conditions in a nuanced way. Therefore, the Gaussian function ensures optimal system performance when using the root mean square sensitivity metric (Figure 4).

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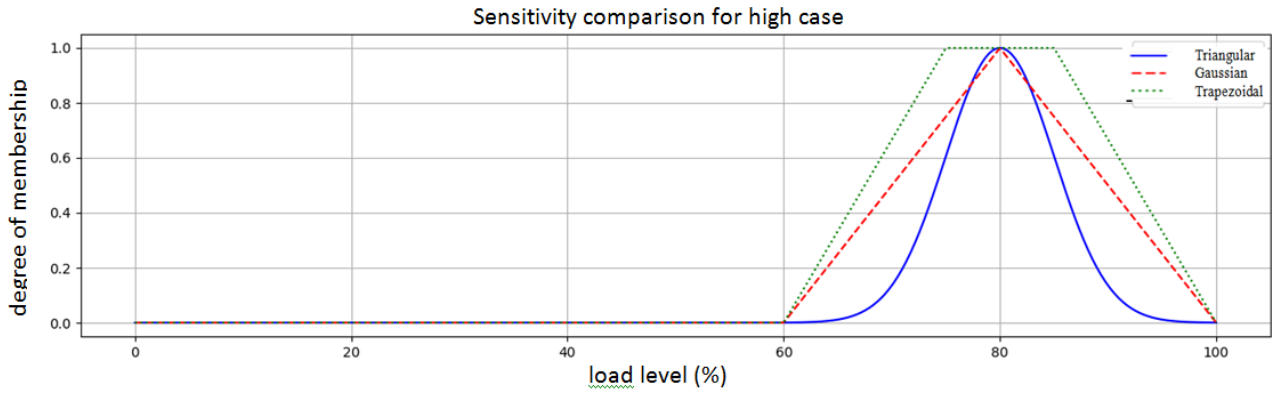

Figure 4. Graphical representation of sensitivity for the high case

As can be seen from the graph, the Gaussian function provides the lowest RMS sensitivity at high load conditions, maximizing the system's forecasting accuracy. The sensitivity graph of the Gaussian function shows how the system responds to changes and provides forecast accuracy. The Triangle and Trapezoid functions, on the other hand, show high RMS sensitivity, limiting the system's rapid responses and reducing forecast accuracy.

According to the experimental results, the Gaussian function provides high forecast accuracy and shows the lowest RMS sensitivity. These results indicate that the Gaussian function is the most effective in optimal forecasting and nuanced evaluation of the system under high load conditions. The experimental results are presented in Table 4 below.

Table 4

Results of comparison of membership functions based on S_{RMS} metrics

	Function	S_{RMS}	Forecast (%)
1	Gaussian	0.092 ± 0.004	93.7 ± 0.8
2	Triangular	0.157 ± 0.006	84.2 ± 1.2
3	Trapezoidal	0.121 ± 0.005	88.9 ± 1.1

The Gaussian function has the lowest RMS sensitivity and high forecast accuracy. This ensures that the system performs well in forecasting and accurately estimating the load condition. The triangular function, on the other hand, has high sensitivity but lower forecast accuracy, which affects the system's ability to respond quickly and accurately under load. The trapezoidal function provides moderate sensitivity and forecast accuracy, but it cannot maximize the system's performance under load.

Summary

In the research, the optimal selection of relevance functions for three different cases and their provision of sensitivity in a combinatorial approach are important. For each case, there are specific sensitivity levels of relevance functions, and based on their combination, it is necessary to control and optimize the overall sensitivity of the system. The sensitivity levels of each feature have a significant impact on the distribution of requests in the system, which plays an important role in effectively managing the decision-making process.

1. A conceptual model based on priority criteria has been developed to implement management taking into account the individual characteristics of the server when dynamically distributing requests. The proposed model consists of several functional stages and is able to activate different mechanisms depending on the differences in the state of the servers. In cases where the server characteristics (computing power, load level, response time) are the same, the system distributes requests based on the classic Round Robin algorithm. However, in cases where there are differences in the parameters of the servers, each parameter is represented as a linguistic variable, a priority weight is formed based on fuzzy logic, and requests are distributed based on these estimates. The model also includes separate modules that optimize the rule base using a genetic algorithm and provide adaptive management based on priority indicators. The proposed approach allows for flexible assessment of the server state in real time, dynamic decision-

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making in relation to the situation, and rational use of resources[6].

2. Linguistic modeling of server parameters made it possible to evaluate the system state in a vague but semantically consistent way. At the fuzzification stage, each parameter - computing power, load level, and response time - was expressed in three main linguistic categories ("low", "medium", "high"). During the study, it was found that applying a single type of membership function to all input parameters reduces the sensitivity of the system and limits the accuracy of the decision. Accordingly, an approach was developed to combine triangular, trapezoidal, and Gaussian shapes based on the semantics of the parameters. This approach significantly increased the degree of differentiation of fuzzy system decisions relative to the state. As a result, the combined selection of membership functions based on the semantic properties of the fuzzy logic system made it possible to provide high sensitivity and discriminativeness.

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